Questions in the Distribution of Prime Numbers

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Prime Numbers General Facts:

- A rational prime number is any number >0, who's only proper divisors are itself, ± 1, and multiples of itself by ± 1.
  e.g.: 2 , 3

- All integers can be uniquely expressed as the product of prime numbers
  e.g. : $100=2^2 * 5^2$

Fundamental Questions

- How do you decide if a new number is prime?
- Given any number how do you factor it into a product of prime numbers?

How do you decide if a new number is prime?

Is there some number (p) that divides the "prime"(n)?

e.g. 299

P | 299

We test , does:

2 | 299 (no)

3 | 299 (no)

5 | 299 (no)

- Up to what value of P must we check to see if 299 (n) is prime?

How do you decide if a number is prime?

- $299 = p * d$

  $d \geq p$

$p * d \geq p * p$

$299 \geq p^2$

$p \leq \sqrt{299}$ (or generically, we can say $p \leq \sqrt{n}$)

$p \leq 17$, so we continue

7 | 299 (no)

11 | 299 (no)

13 | 299 (yes, 13*23 = 299)

Sieve of Eratosthenes

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20

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Pari/GP

- Pari is a powerful command based symbolic calculation program
- Pari also has a programmable interface, so we are able to script new functions into the software

```
PARI
Type ? for help, \q to quit.
Type ?12 for how to get moral (and possibly technical) support.

parisize = 4000000, primelimit = 500000
(22:05) gp > read("updated.gp")
(22:05) gp > countmod4(100)
The number of primes <100 = 1 mod 4 is 11
The number of primes <100 = 3 mod 4 is 13
The percentage of primes <100 = 1 mod 4 is 0.4400000000000000000000000000
The percentage of primes <100 = 3 mod 4 is 0.5200000000000000000000000000
(22:05) gp > printprime(50)
2
3
5
7
11
13
17
19
23
29
31
37
41
43
47
(22:05) gp > _
```

Prime Number Races

- Using Modular Arithmetic, the prime numbers were divided into two columns (1Mod4, & 3Mod4)

1Mod4	3Mod4
5	3
13	7
17	11
29	19
	23

Prime Number Races

- As far as this race goes, it seems that 3Mod4 is Always in the lead. In fact, as the primes approach ∞, 3Mod4 wins approximately 99.59% of the time.
- This was also done for Mod3, Mod5, and Mod6
- The “losers” of the prime number races are ≡ (any square number mod “n” in the race of mod “n”)

  e.g. $3^2 = 9$

  $9(4) \equiv 1(4)$

Gaussian Prime Numbers

- Gaussian Integers are numbers in the form of a+bi, where a & b are both integers. (also known as complex numbers)

e.g.: 2+3i

6-8i

- A Gaussian Integer is a Gaussian Prime if it's only proper divisors are ± 1, ± i, and multiples of itself by ± 1, ± i.

e.g.: 1+2i = (-1)(-1-2i)

(-i)(2-i)   1+2i is a Gaussian Prime

2+2i = 2i(1-i)   2+2i is Not a Gaussian prime.

Gaussian Prime Numbers

- Sometimes a rational prime is also a Gaussian Prime, and sometimes it is <u>NOT</u>.

NOT:

e.g.: $5 = (2+i)(2-i)$

$= 4 + 1$; 5 is not a Gaussian Prime.

However 7 is a Gaussian prime, since it cannot be factored into the form $(a + bi)(a - bi)$, or $a^2 + b^2$

*Note: Rational numbers can only be factored in conjugate a + bi form.

Gaussian Prime Numbers

- If a rational prime cannot be written in the form $a^2 + b^2$ then it is also a Gaussian prime.

- However, if the rational prime can be written in the form $a^2 + b^2$, then factoring it will give you Gaussian primes in the form (a±bi) and (b±ai)

Gaussian Prime Numbers

- We arranged the Gaussian primes in order of magnitude based on their “norms”. This is calculated by multiplying the prime by it’s conjugate.

  e.g.: 1+2i

  (1+2i)(1-2i)=5

  When dealing with Gaussian Primes, they cannot be broken up into categories by modular arithmetic, as the rational primes were. These are broken into angle categories.

  The “angle” of a Gaussian prime is calculated by arctan(b/a)

Credits

- Professor Gautam Chinta
- CCNY
- HCS staff
- Dr. Sat Battacharya
- The Audience!

THANK YOU!